The Method of Lines for the Analysis of Planar Waveguides with Finite Metallization Thickness

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***Abstract*—The method of lines has been applied to many different planar waveguide structures, but up to now only infinitely thin metallizations have been considered. In this paper it will be shown how the method can be extended for the analysis of waveguides with finite metallization thickness. The results for microstrip and finline are presented and compared with those of other authors.**

I. Introduction

In most analyses of planar waveguides the metallizations are assumed to be infinitely thin. With the decreasing dimensions of monolithic integrated microwave circuits, this assumption must be discarded, because both the cross-sectional dimensions and the wavelength are of the same order of magnitude. Microstrip lines and finlines with finite metallization thickness have been investigated with various procedures, for example, with full-wave-analyses [1]–[3], with the conformal mapping technique [4], [5], and with the point-matching method [6]. In the present paper the method of lines (MoL) [7], [8] is extended to calculate the properties of the wave propagation in shielded microstrip lines and finlines with finite metallization thickness. The extension is straightforward, because there is no change in the principles of the MoL in inserting the additional layers necessary to consider the finite thick metallizations.

Contrary to the previous investigations where the layers have been completely filled with dielectric, the additional layers are split into regions of dielectric and metallization respectively. The ranges of the metallization thicknesses are not restricted in the numerical treatment. Therefore metallizations substantially thicker than in the point-matching method [6] are computed. For small thicknesses it is possible to obtain the dispersion constant with only one calculation when choosing a certain discretization which depends on an optimal edge parameter p_{opt}. In this case, the advantage of the MoL is particularly clear because only a small number of discretization lines are necessary in the calculation. Hence the computing time is very small.

II. Theory

The investigated shielded waveguide (Fig. 1) includes layers which are filled only with dielectrics as well as a layer with alternating dielectric and metallization regions. An extension of the analysis for structures with metallizations in more than one layer is easily possible. The analysis is described in detail in [7] and therefore is not repeated here. Just the essential parts of the method and its extension are summarized in the following steps.

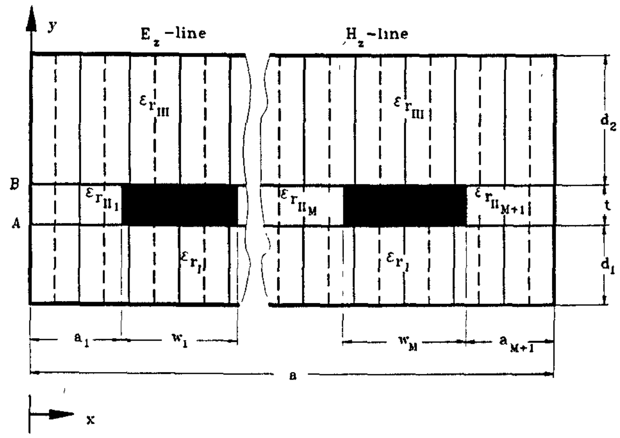

Fig. 1. Waveguide structure with finite thickness of the metallization t.

1) The electromagnetic fields are calculated from the independent field components E_z and H_z. The other field components are derived from these by using Maxwell's equations.

2) The fields, the field equations, and the wave equations (Helmholtz equations for E_z and H_z) of each layer, as well as those of the slots in layer II, are discretized in one direction (along the x coordinate in Fig. 1).

3) The resulting differential equations are decoupled by several suitable mathematical transformations and then analytically solved along the y coordinate. Different transformation matrices are applied owing to the different line numbers in layers I and III on the one hand and in the slots of layer II on the other hand. The relations between the transformed tangential fields at the interfaces to the adjacent layers are now calculated with those solutions and yield

$$\begin{bmatrix} \overline{H}_A^{I} \\ \overline{H}_B^{III} \end{bmatrix} = \begin{bmatrix} Y_I & \\ & Y_{III} \end{bmatrix} \begin{bmatrix} \overline{E}_A^{I} \\ \overline{E}_B^{III} \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} \overline{H}_A^{II_i} \\ \overline{H}_B^{II_i} \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11}^{II_i} & Y_{12}^{II_i} \\ Y_{21}^{II_i} & Y_{22}^{II_i} \end{bmatrix}}_{Y^{II_i}} \begin{bmatrix} \overline{E}_A^{II_i} \\ \overline{E}_B^{II_i} \end{bmatrix}. \tag{2}$$

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4) To prepare the field matching in the spatial domain, it is necessary to transform back (1) and (2) with the corresponding transformation matrices. The inverse transform of (1) is then split into two systems of equations, (3) and (4), where system (3) describes the relations between the fields at the interfaces A and B of the slots in layer II, and system (4) describes the relations between the currents $\boldsymbol{J}$ on the strips and the fields at the interfaces A and B of the slots:

$$\boldsymbol{H}_{\text{Slots}}^{\text{I, III}} = f\left(\boldsymbol{E}_{\text{Slots}}^{\text{I, III}}\right) \tag{3}$$

$$\boldsymbol{J}_{\text{Strips}}^{\text{I, III}} = f\left(\boldsymbol{E}_{\text{Slots}}^{\text{I, III}}\right). \tag{4}$$

In the following steps only (3) is used. To obtain this equation the inverse transform of (1) is done with the reduced transformation matrices $T_{\mathrm{I}}^{r} = T_{\mathrm{III}}^{r}$ (6), which are rectangular and consist of those rows of the complete transformation matrices $T_{\mathrm{I,\ III}}$ (5) which correspond to the discretization lines crossing any of the slots of layer II.

$$T_{\mathrm{I}} = \begin{bmatrix} T_{DN}^{\mathrm{I}} & \\ & T_{ND}^{\mathrm{I}} \end{bmatrix} = \begin{bmatrix} \begin{array}{|c|}\hline \text{slot } 1 \\ \hline \text{strip } 1 \\ \hline \text{slot } 2 \\ \hline \vdots \\ \hline \text{strip } M \\ \hline \text{slot } M+1 \\ \hline \end{array} & \\ & \begin{array}{|c|}\hline \text{slot } 1 \\ \hline \text{strip } 1 \\ \hline \text{slot } 2 \\ \hline \vdots \\ \hline \text{strip } M \\ \hline \text{slot } M+1 \\ \hline \end{array} \end{bmatrix}. \tag{5}$$

$$T_{\mathrm{I}}^{r} = \begin{bmatrix} T_{DN}^{\mathrm{I}r} & \\ & T_{ND}^{\mathrm{I}r} \end{bmatrix} = \begin{bmatrix} \begin{array}{|c|}\hline \text{slot } 1 \\ \hline \text{slot } 2 \\ \hline \vdots \\ \hline \text{slot } M+1 \\ \hline \end{array} & \\ & \begin{array}{|c|}\hline \text{slot } 1 \\ \hline \text{slot } 2 \\ \hline \vdots \\ \hline \text{slot } M+1 \\ \hline \end{array} \end{bmatrix} \tag{6}$$

Hence from (1) it follows that

$$\begin{bmatrix} \boldsymbol{H}_{A,\text{Slots}} \\ \boldsymbol{H}_{B,\text{Slots}} \end{bmatrix} = \underbrace{\begin{bmatrix} T_{\mathrm{I}}^{r} & \\ & T_{\mathrm{I}}^{r} \end{bmatrix}}_{\tilde{T}_{\mathrm{I}}^{r}} \underbrace{\begin{bmatrix} Y^{\mathrm{I}} & \\ & Y^{\mathrm{III}} \end{bmatrix}}_{\tilde{Y}^{\mathrm{I,\,III}}} \underbrace{\begin{bmatrix} T_{\mathrm{I}}^{r} & \\ & T_{\mathrm{I}}^{r} \end{bmatrix}^{t}}_{\tilde{T}_{\mathrm{I}}^{rt}} \begin{bmatrix} \boldsymbol{E}_{A,\text{Slots}} \\ \boldsymbol{E}_{B,\text{Slots}} \end{bmatrix}. \tag{7}$$

Analogously all field relations in the slots of layer II are transformed back and the resulting $i = 1,\ M+1$ systems of equations is summarized in (8) corresponding to the

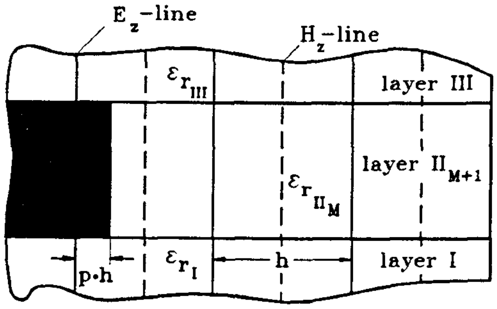

Fig. 2. Position of the discretization lines in the environment of a metallization edge.

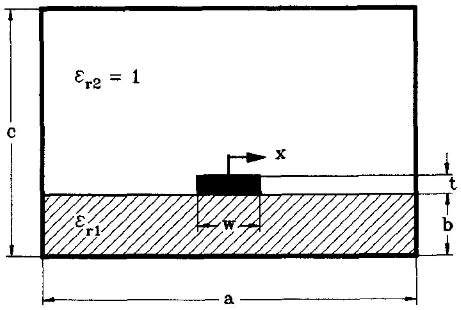

Fig. 3. Cross section of the investigated microstrip line.

structure in (7):

$$\begin{bmatrix} \boldsymbol{H}_{A,\text{Slots}} \\ \boldsymbol{H}_{B,\text{Slots}} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{T}_{\mathrm{II}} & \\ & \boldsymbol{T}_{\mathrm{II}} \end{bmatrix}}_{\tilde{T}_{\mathrm{II}}} \underbrace{\begin{bmatrix} Y_1^{\mathrm{II}} & Y_2^{\mathrm{II}} \\ Y_2^{\mathrm{II}} & Y_1^{\mathrm{II}} \end{bmatrix}}_{\tilde{Y}^{\mathrm{II}}} \underbrace{\begin{bmatrix} \boldsymbol{T}_{\mathrm{II}} & \\ & \boldsymbol{T}_{\mathrm{II}} \end{bmatrix}^{t}}_{\tilde{T}_{\mathrm{II}}^{t}} \underbrace{\begin{bmatrix} \boldsymbol{E}_{A,\text{Slots}} \\ \boldsymbol{E}_{B,\text{Slots}} \end{bmatrix}}_{\boldsymbol{E}_{AB,\text{Slots}}} \tag{8}$$

with $\boldsymbol{T}_{\mathrm{II}} = \mathrm{Diag}(T_{\mathrm{II}_i})$; $i = 1, M+1$.

5) The field matching in the spatial domain follows with (7) and (8) and yield the homogeneous equation

$$\left(\tilde{\boldsymbol{T}}_{\mathrm{II}}\tilde{\boldsymbol{Y}}^{\mathrm{II}}\tilde{\boldsymbol{T}}_{\mathrm{II}}^{t} - \tilde{\boldsymbol{T}}_{\mathrm{I}}^{r}\tilde{\boldsymbol{Y}}^{\mathrm{I,\,III}}\tilde{\boldsymbol{T}}_{\mathrm{I}}^{rt}\right)\boldsymbol{E}_{AB,\text{Slots}} = 0. \tag{9}$$

From the nontrivial solution of this homogeneous equation the electrical characteristics of the wave propagation are calculated.

III. Results

The convergence behavior of the calculated results depends on the position of the metallization edges in relation to the adjacent discretization lines, described by the edge parameter p in Fig. 2. Because of the discretization on two families of lines the position of the edge in Fig. 2 is restricted to the interval from $p = 0$ (E_z line) $\cdots$ $p < 0.5$ (up to, but not including, the following H_z line). For infinite thickness an optimal convergence is obtained when positioning the edge at $p = 0.25$ [8]. Fig. 3 shows the cross section of the investigated microstrip line and (a) and (b) of Fig. 4 show the corresponding convergence behavior of the dispersion constant for varying edge parameters p and different thicknesses ($t = 1\ \mu$m, $10\ \mu$m) of the metallization. For the finline structure (Fig. 5), the convergence investigation is shown in parts (a) and (b) of Fig. 6 even for varying edge parameters p and different thicknesses ($t = 1\ \mu$m, $15\ \mu$m) of the fins.

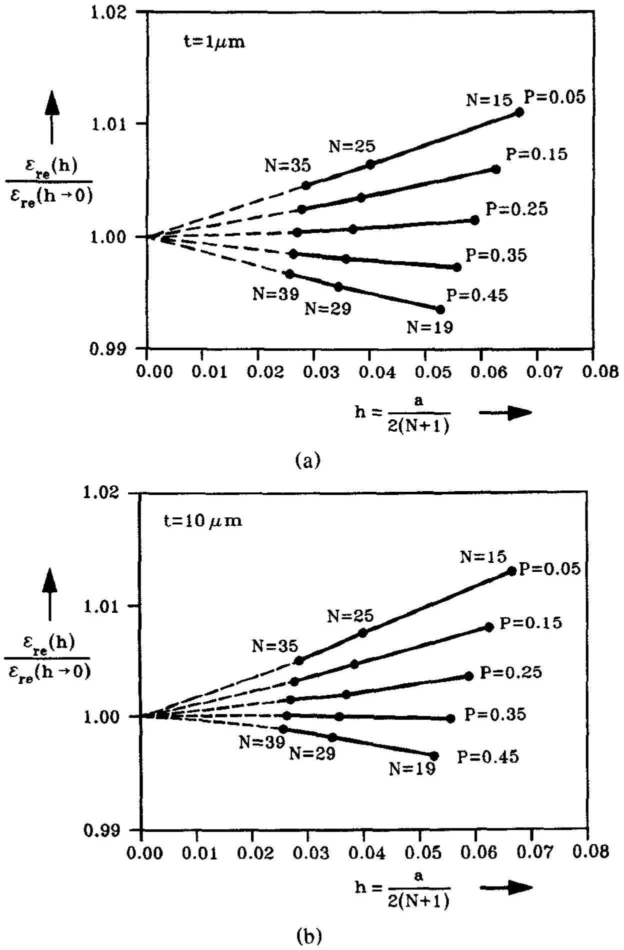

Fig. 4. (a) Convergence of the dispersion constant versus the discretization distance (microstrip line $a = 6.0$ mm, $w = 0.6$ mm, $b = 0.635$ mm, $c = 100.0$ mm, $\epsilon_{r1} = 10$, $t = 1$ μm). (b) Convergence of the dispersion constant versus the discretization distance (microstrip line, see Fig. 4(a) with $t = 10$ μm).

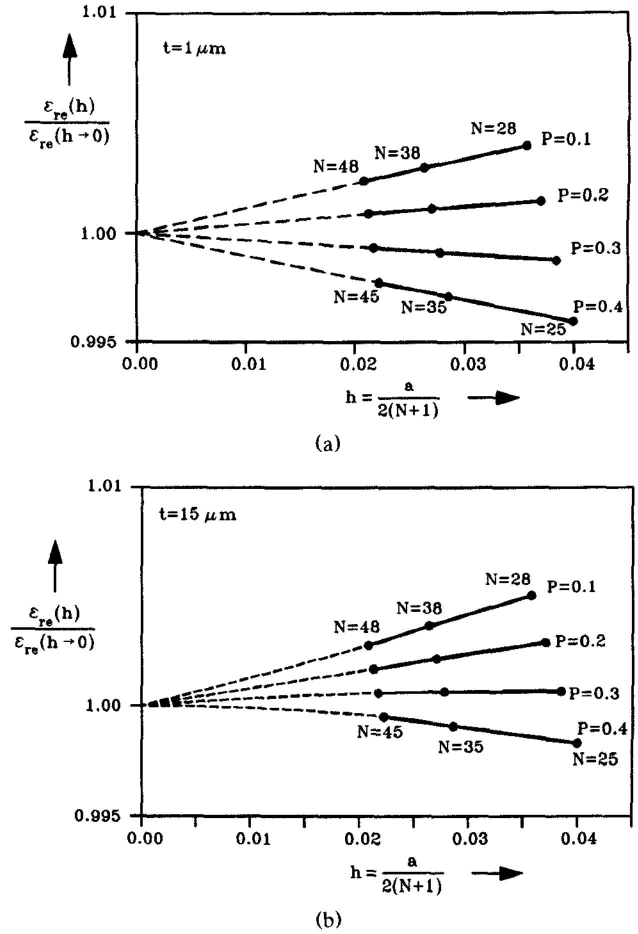

Fig. 6. (a) Convergence of the dispersion constant versus the discretization distance (finline $a = 6.0$ mm, $s = 0.6$ mm, $b = d = 5.5$ mm, $c = 1.0$ mm, $\epsilon_{r1} = 5$, $\epsilon_{r2} = 9.7$, $t = 1$ μm). (b) Convergence of the dispersion constant versus the discretization distance (finline, see Fig. 6(a) with $t = 15$ μm)

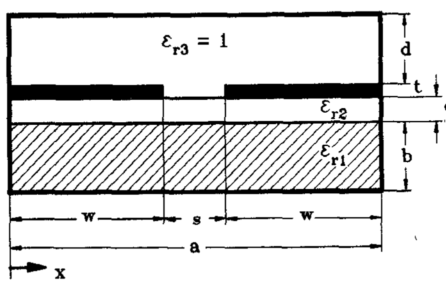

Fig. 5. Cross section of the investigated finline.

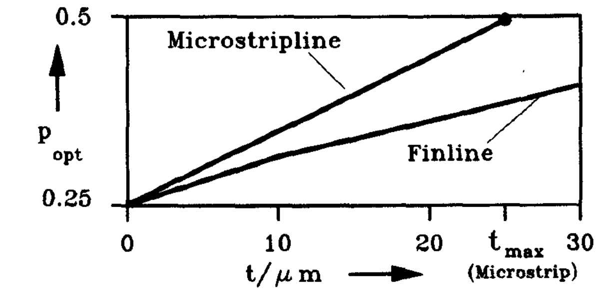

Fig. 7. Optimal edge parameter, p_{opt}, versus metallization thickness t.

The slopes of the penciled curves reach from positive to negative values, so that a curve with a slope of nearly zero can be found. The corresponding edge parameter is then the optimal edge parameter, p_{opt}. From Figs. 4(a) and (b) and 6(a) and (b) and further convergence calculations, optimal edge parameters, p_{opt}, are obtained and are shown in Fig. 7 depending on the thickness of the metallizations. Using this optimal edge parameter, p_{opt}, in the calculation yields a convergence curve for the dispersion constant ϵ_{re}, which runs nearly at a constant value. Hence only one computation with a small line number is necessary to obtain an accurate value of ϵ_{re}, which is very close to the exact dispersion constant $\epsilon_{re0} = \epsilon_{re}(h \to 0)$. As the reason for this behavior, it is assumed that the fields at the edges of the metallizations, especially of thin ones, are better approximated by considering the optimal edge parameter, p_{opt}, in the calculation.

With increasing thickness the family of curves, compared with the extrapolated ϵ_{re0}, is shifted more and more upward, and the curves become more bent with increasing p. When exceeding a certain thickness t_{max} all calculated points are greater than ϵ_{re0} and an optimal edge parameter p_{opt} cannot be determined. Nevertheless structures with a greater thick-

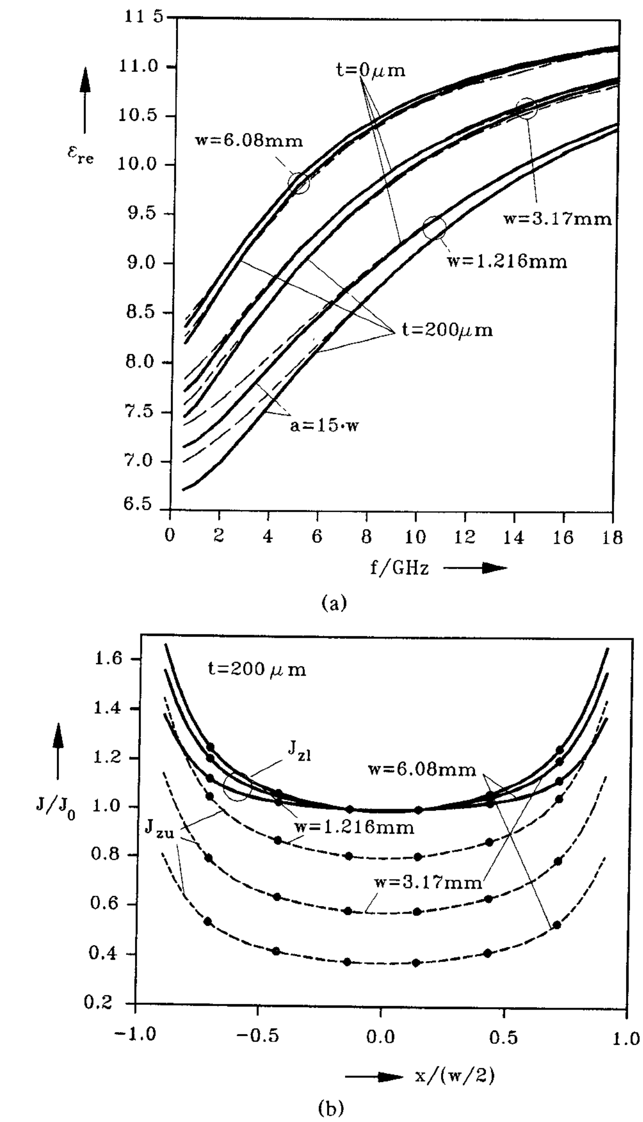

Fig. 8. (a) Dispersion of a microstrip line ($a = 9 \cdot w$, $b = 3.04$ mm, $c = 100.0$ mm, $\epsilon_{r1} = 11.7$) ---: results from [5] (open structure). (b) Current density on the strip (subscript l: lower side of the strip, subscript u: upper side of the strip, $J_0 = J_{zl}(x = 0)$, $f = 0.5$ GHz).

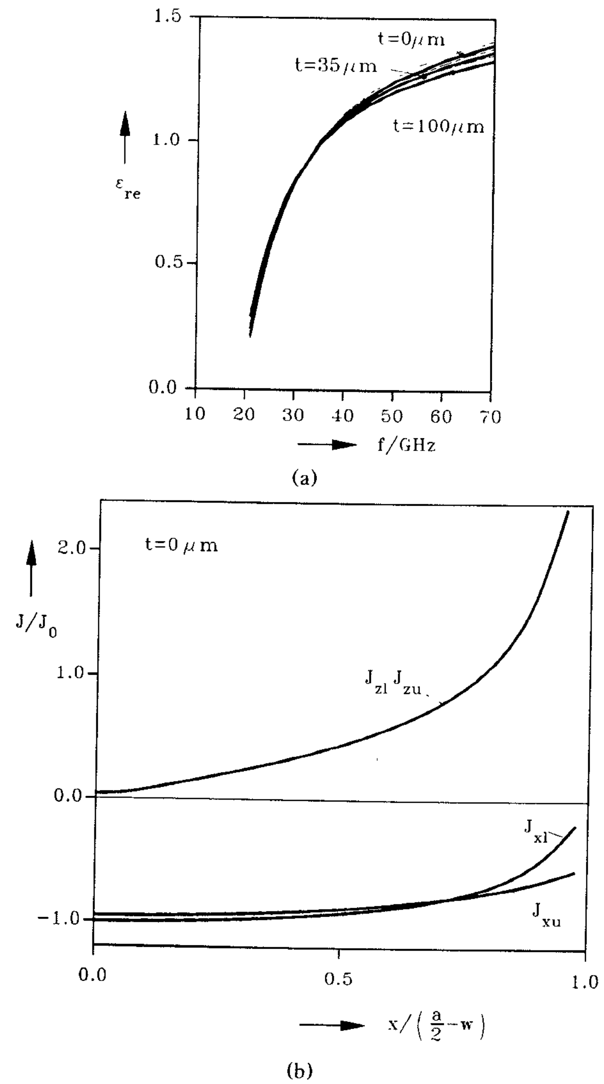

Fig. 9. (a) Dispersion of a finline ($a = 2.3876$ mm, $s = 0.2a$, $b = 2.2606$ mm, $c = 0.127$ mm, $d = 2.3876$ mm $- t$, $\epsilon_{r1} = 1.0$, $\epsilon_{r2} = 3.8$) ---: results from [3]. (b) Current density (subscript l: lower side of the strip, subscript u: upper side of the strip, $J_0 = J_{xl}(x = 0)$, $f = 21$ GHz).

ness of metallization were calculated and therefore convergence investigations became necessary. In these calculations the edge parameter p has been chosen to be less than 0.1 to obtain monotonically decreasing convergence curves, which are easy to extrapolate.

Fig. 8(a) shows the dispersion constants of a microstrip line with varying thickness t and width w of the strip, compared with results from [5]. The deviations of the dispersion constants at low frequences in this figure are explained by the effect of the shielding. Fig. 8(b) represents the current densities J_z on the surface of the strip of the corresponding microstrip lines of Fig. 8(a). Due to the fast rising curve of the currents J_z in the regions close to the edges, an accurate calculation of the current distribution in these regions is not possible.The deviation of the computed current from the exact current increases, particularly very close the edges. Hence in Fig. 8(b) these regions are not considered.

Parts (a) and (b) of Fig. 9 show the dispersion constant of a finline with varying thickness t of the metallization, compared with results from [3], as well as the current density.

IV. Conclusions

The method of lines is well suited to calculate waveguide structures even with finite metallization thickness. The normally used range of metallization thickness is not a limit for the method. Particularly when calculating small or moderate thicknesses, it is possible to derive the dispersion constant with only one computed result. When using the optimal edge parameter, p_{opt}, for that computation the deviation from the exact dispersion constant is less than 0.5%. The advantage of the method of lines is that only small line numbers are necessary. Hence the computing time is very small.

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